WHAT IS CLAIMED IS:

1. A method for computing all occurrences of a compound event from occurrences of primitive events where the compound event is a defined combination of the primitive events, the method comprising the steps of:

(a) defining primitive event types;

(b) defining combinations of the primitive event types as a compound event type;

(c) inputting the primitive event occurrences, such occurrences being specified as the set of temporal intervals over which a given primitive event type is true; and

(d) computing the compound event occurrences, such occurrences being specified as the set of temporal intervals over which the compound event type is true, wherein the sets of temporal intervals in steps (c) and (d) are specified as smaller sets of spanning intervals, each spanning interval representing a set of intervals.

2. The method according to claim 1, wherein the spanning intervals take the form ${}_{\alpha}[{}_{\gamma}[i,j]_{\delta},{}_{\in}[k,l]_{\zeta}]_{\beta}$, where $\alpha,\beta,\gamma,\delta,\in$, and ζ are Boolean values, i,j,k, and l are real numbers, ${}_{\alpha}[{}_{\gamma}[i,j]_{\delta},{}_{\in}[k,l]_{\zeta}]_{\beta}$ represents the set of all intervals ${}_{\alpha}[p,q]_{\beta}$ where $i \leq_{\gamma} p \leq_{\delta} j$ and $k \leq_{\in} q \leq_{\zeta} l$, ${}_{\alpha}[p,q]_{\beta}$ represents the set of all points r, where $p \leq_{\alpha} r \leq_{\beta} q$, and $x \leq_{\theta} y$ means $x \leq y$ when θ is true and $x < y$ when θ is false.

3. The method according to claim 2, wherein the compound event type in step (b) is specified as an expression in temporal logic.

4. The method according to claim 3, wherein the temporal logic expressions are constructed using the logical connectives $\forall, \exists, \vee, \wedge_R, \Diamond_R$, and $\neg$, where R ranges over sets of relations between one-dimensional intervals.

5. The method according to claim 4, wherein the relations are =, <, >, m, mi, o, oi, s, si, f, fi, d, and di.

6. The method according to claim 5, wherein the compound event occurrences are computed using the following set of equations:

$$
\begin{array}{rcl}
\varepsilon(M,p(c_1,\ldots,c_n)) & \triangleq & \{\mathbf{i} \mid p(c_1,\ldots,c_n)@\mathbf{i}\in M\} \\
\varepsilon(M,\Phi\vee\Psi) & \triangleq & \varepsilon(M,\Phi)\cup\varepsilon(M,\Psi) \\
\varepsilon(M,\forall x\Phi) & \triangleq & \bigcup_{\mathbf{i}_1\in\varepsilon(M,\Phi[x:=c_1])}\cdots\bigcup_{\mathbf{i}_n\in\varepsilon(M,\Phi[x:=c_n])}\mathbf{i}_1\cap\cdots\cap\mathbf{i}_n \\
 & & \text{where } C(M)=\{c_1,\ldots,c_n\} \\
\varepsilon(M,\exists x\Phi) & \triangleq & \bigcup_{c\in C(M)}\varepsilon(M,\Phi[x:=c]) \\
\varepsilon(M,\neg\Phi) & \triangleq & \bigcup_{\mathbf{i}'_1\in\neg\mathbf{i}_1}\cdots\bigcup_{\mathbf{i}'_n\in\neg\mathbf{i}_n}\mathbf{i}'_1\cap\cdots\cap\mathbf{i}'_n \\
 & & \text{where } \varepsilon(M,\Phi)=\{\mathbf{i}_1,\ldots,\mathbf{i}_n\} \\
\varepsilon(M,\Phi\wedge_R\Psi) & \triangleq & \bigcup_{\mathbf{i}\in\varepsilon(M,\Phi)}\bigcup_{\mathbf{j}\in\varepsilon(M,\Psi)}\bigcup_{r\in R}\mathcal{I}(\mathbf{i},r,\mathbf{j}) \\
\varepsilon(M,\Diamond_R\Phi) & \triangleq & \bigcup_{\mathbf{i}\in\varepsilon(M,\Phi)}\bigcup_{r\in R}\mathfrak{D}(r,\mathbf{i})
\end{array}
$$

where,

$$
\langle{}_{\alpha}[_{\gamma}[i,j]_{\delta,\in}[k,l]_{\zeta}]_{\beta}\rangle\triangleq
\begin{cases}
\{{}_{\alpha}[_{\gamma'}[i,j']_{\delta',\in'}[k',l]_{\zeta'}]_{\beta}\} & \\
\text{where } j'=\min(j,l) & \\
\qquad k'=\max(k,i) & \\
\qquad \gamma'=\gamma\wedge i\neq-\infty & \\
\qquad \delta'=\delta\wedge\min(j,l)\neq\infty\wedge(j<l\vee\zeta\wedge\alpha\wedge\beta) & \\
\qquad \in'=\in\wedge\max(k,i)\neq-\infty\wedge(k>i\vee\gamma\wedge\beta\wedge\alpha) & \\
\qquad \zeta'=\zeta\wedge l\neq\infty & \\
\text{when } i\leq j'\wedge k'\leq l\wedge & \\
\qquad [i=j'\rightarrow(\gamma'\wedge\delta')]\wedge[k'=l\rightarrow(\in'\wedge\zeta')]\wedge & \\
\qquad [i=l\rightarrow(\alpha\wedge\beta)]\wedge & \\
\qquad i\neq\infty\wedge j'\neq-\infty\wedge k'\neq\infty\wedge l\neq-\infty & \\
\{\} \qquad \text{otherwise} &
\end{cases}
$$

$$
{}_{\alpha_1}[_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \cap_{\alpha_2} [_{\gamma_2}[i_2, j_2]_{\delta_2}, {}_{\in_2}[k_2, l_2]_{\zeta_2}]_{\beta_2} \triangleq
$$

$$
\langle_{\alpha_1}[_{\gamma}[\max(i_1, i_2), \min(j_1, j_2)]_{\delta}, {}_{\in}[\max(k_1, k_2), \min(l_1, l_2)]_{\zeta}]_{\beta_1}\rangle
$$

$$
\text{where } \gamma = \begin{cases} \gamma_1 & i_1 > i_2 \\ \gamma_1 \wedge \gamma_2 & i_1 = i_2 \\ \gamma_2 & i_1 < i_2 \end{cases}
$$

$$
\delta = \begin{cases} \delta_1 & j_1 < j_2 \\ \delta_1 \wedge \delta_2 & j_1 = j_2 \\ \delta_2 & j_1 > j_2 \end{cases}
$$

$$
\in = \begin{cases} \in_1 & k_1 > k_2 \\ \in_1 \wedge \in_2 & k_1 = k_2 \\ \in_2 & k_1 < k_2 \end{cases}
$$

$$
\zeta = \begin{cases} \zeta_1 & l_1 < l_2 \\ \zeta_1 \wedge \zeta_2 & l_1 = l_2 \\ \zeta_2 & l_1 > l_2 \end{cases}
$$

when $\alpha_1 = \alpha_2 \wedge \beta_1 = \beta_2$

{} otherwise

$$
\neg_{\alpha}[_{\gamma}[i, j]_{\delta}, {}_{\in}[k, l]_{\zeta}]_{\beta} \triangleq \begin{pmatrix} \langle_{\alpha}[_{T}[-\infty, \infty]_{T}, {}_{T}[-\infty, k]_{\neg\in}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{T}[-\infty, \infty]_{T}, {}_{\neg\zeta}[l, \infty]_{T}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{T}[-\infty, i]_{\neg\gamma}, {}_{T}[-\infty, \infty]_{T}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{\neg\delta}[j, \infty]_{T}, {}_{T}[-\infty, \infty]_{T}]_{\beta}\rangle \cup \\ \langle_{\neg\alpha}[_{T}[-\infty, \infty]_{T}, {}_{T}[-\infty, \infty]_{T}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{T}[-\infty, \infty]_{T}, {}_{T}[-\infty, \infty]_{T}]_{\neg\beta}\rangle \cup \\ \langle_{\neg\alpha}[_{T}[-\infty, \infty]_{T}, {}_{T}[-\infty, \infty]_{T}]_{\neg\beta}\rangle \end{pmatrix}
$$

$$\textsc{Span}\left({}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}, {}_{\alpha_2}[{}_{\gamma_2}[i_2, j_2]_{\delta_2}, {}_{\in_2}[k_2, l_2]_{\zeta_2}]_{\beta_2}\right) \triangleq$$

$$\begin{pmatrix} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j]_{\delta}, {}_{\in}[k, l_1]_{\zeta_1}]_{\beta_1} \rangle \cup \\ \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j]_{\delta}, {}_{\in}[k, l_2]_{\zeta_2}]_{\beta_2} \rangle \cup \\ \langle {}_{\alpha_2}[{}_{\gamma_2}[i_2, j]_{\delta}, {}_{\in}[k, l_1]_{\zeta_1}]_{\beta_1} \rangle \cup \\ \langle {}_{\alpha_2}[{}_{\gamma_2}[i_2, j]_{\delta}, {}_{\in}[k, l_2]_{\zeta_2}]_{\beta_2} \rangle \end{pmatrix}$$

where $j = \min(j_1, j_2)$

$$k = \max(k_1, k_2)$$

$$\delta = [(\delta_1 \wedge j_1 \leq j_2) \vee (\delta_2 \wedge j_1 \geq j_2)]$$

$$\in = [(\in_1 \wedge k_1 \geq k_2) \vee (\in_2 \wedge k_1 \leq k_2)]$$

$$\mathfrak{D}(=, \mathbf{i}) \triangleq \{\mathbf{i}\}$$

$$\mathfrak{D}(<, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\neg\beta_1 \wedge \neg\alpha_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}, {}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(>, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}, {}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \neg\beta_2 \wedge \delta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{m}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\neg\beta_1}[{}_{\in_1}[k_1, l_1]_{\zeta_1}, {}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{mi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}, {}_{\gamma_1}[i_1, j_1]_{\delta_1}]_{\neg\alpha_1} \rangle$$

$$\mathfrak{D}(\mathsf{o}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \alpha_2 \wedge \zeta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{oi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\alpha_1 \wedge \beta_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{s}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{si}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathrm{T}, \mathrm{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\mathrm{T}}[-\infty, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{f}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2 \in \{\mathrm{T}, \mathrm{F}\}} \langle {}_{\alpha_2}[{}_{\mathrm{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \rangle$$

$$\mathfrak{D}(\mathsf{fi},{}_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\epsilon_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2\in\{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[_{\alpha_1\wedge\neg\alpha_2\wedge\gamma_1}[i_1,\infty]_{\mathbf{T}},_{\epsilon_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}\rangle$$

$$\mathfrak{D}(\mathsf{d},{}_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\epsilon_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2,\beta_2\in\{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[_{\mathbf{T}}[-\infty,j_1]_{\neg\alpha_1\wedge\alpha_2\wedge\delta_1},_{\neg\beta_1\wedge\beta_2\wedge\epsilon_1}[k_1,\infty]_{\mathbf{T}}]_{\beta_2}\rangle$$

$$\mathfrak{D}(\mathsf{di},{}_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\epsilon_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2,\beta_2\in\{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[_{\alpha_1\wedge\neg\alpha_2\wedge\gamma_1}[i_1,\infty]_{\mathbf{T}},_{\mathbf{T}}[-\infty,l_1]_{\beta_1\wedge\neg\beta_2\wedge\zeta_1}]_{\beta_2}\rangle$$

and, $$\mathcal{I}(\mathbf{i},r,\mathbf{j}) \triangleq \bigcup_{\mathbf{i}'\in\mathfrak{D}(r^{-1},\mathbf{j})} \bigcup_{\mathbf{i}''\in\mathbf{i}'\cap\mathbf{i}} \bigcup_{\mathbf{j}'\in\mathfrak{D}(r,\mathbf{i})} \bigcup_{\mathbf{j}''\in\mathbf{j}'\cap\mathbf{j}} \textsc{Span}(\mathbf{i}'',\mathbf{j}'').$$

7. A program storage device readable by machine, tangibly embodying a program of instructions executable by machine to perform method steps for computing all occurrences of a compound event from occurrences of primitive events where the compound event is a defined combination of the primitive events, the method comprising the steps of:

(a) defining primitive event types;

(b) defining combinations of the primitive event types as a compound event type;

(c) inputting the primitive event occurrences, such occurrences being specified as the set of temporal intervals over which a given primitive event type is true; and

(d) computing the compound event occurrences, such occurrences being specified as the set of temporal intervals over which the compound event type is true, wherein the sets of temporal intervals in steps (c) and (d) are specified as smaller sets of spanning intervals, each spanning interval representing a set of intervals.

8. The program storage device according to claim 7, wherein the spanning intervals take the form ${}_{\alpha}[_{\gamma}[i,j]_{\delta},_{\epsilon}[k,l]_{\zeta}]_{\beta}$, where $\alpha,\beta,\gamma,\delta,\epsilon,$ and ζ are Boolean values, $i,j,k,$ and l are real numbers, ${}_{\alpha}[_{\gamma}[i,j]_{\delta},_{\epsilon}[k,l]_{\zeta}]_{\beta}$ represents the set of all intervals ${}_{\alpha}[p,q]_{\beta}$ where $i\leq_{\gamma} p\leq_{\delta} j$ and $k\leq_{\epsilon} q\leq_{\zeta} l$, ${}_{\alpha}[p,q]_{\beta}$ represents the set of all

points r where $p \le_\alpha r \le_\beta q$, and $x \le_\theta y$ means $x \le y$ when θ is true and $x < y$ when θ is false.

9. The program storage device according to claim 8, wherein the compound event type in step (b) is specified as an expression in temporal logic.

10. The program storage device according to claim 9, wherein the temporal logic expressions are constructed using the logical connectives $\forall, \exists, \vee, \wedge_R, \Diamond_R$, and $\neg$, where R ranges over sets of relations between one-dimensional intervals.

11. The program storage device according to claim 10, wherein the relations are =, <, >, m, mi, o, oi, s, si, f, fi, d, and di.

12. The method according to claim 11, wherein the compound event occurrences are computed using the following set of equations:

$$
\begin{array}{rcl}
\mathcal{E}(M, p(c_1,\ldots,c_n)) & \triangleq & \{\mathbf{i} \mid p(c_1,\ldots,c_n)@\mathbf{i} \in M\} \\
\mathcal{E}(M, \Phi \vee \Psi) & \triangleq & \mathcal{E}(M,\Phi) \cup \mathcal{E}(M,\Psi) \\
\mathcal{E}(M, \forall x \Phi) & \triangleq & \displaystyle\bigcup_{\mathbf{i}_1 \in \mathcal{E}(M,\Phi[x:=c_1])} \cdots \bigcup_{\mathbf{i}_n \in \mathcal{E}(M,\Phi[x:=c_n])} \mathbf{i}_1 \cap \cdots \cap \mathbf{i}_n \\
 & & \text{where } C(M) = \{c_1,\ldots,c_n\} \\
\mathcal{E}(M, \exists x \Phi) & \triangleq & \displaystyle\bigcup_{c \in C(M)} \mathcal{E}(M, \Phi[x:=c]) \\
\mathcal{E}(M, \neg \Phi) & \triangleq & \displaystyle\bigcup_{\mathbf{i}'_1 \in \neg \mathbf{i}_1} \cdots \bigcup_{\mathbf{i}'_n \in \neg \mathbf{i}_n} \mathbf{i}'_1 \cap \cdots \cap \mathbf{i}'_n \\
 & & \text{where } \mathcal{E}(M,\Phi) = \{\mathbf{i}_1,\ldots,\mathbf{i}_n\} \\
\mathcal{E}(M, \Phi \wedge_R \Psi) & \triangleq & \displaystyle\bigcup_{\mathbf{i} \in \mathcal{E}(M,\Phi)} \bigcup_{\mathbf{j} \in \mathcal{E}(M,\Psi)} \bigcup_{r \in R} \mathcal{I}(\mathbf{i}, r, \mathbf{j}) \\
\mathcal{E}(M, \Diamond_R \Phi) & \triangleq & \displaystyle\bigcup_{\mathbf{i} \in \mathcal{E}(M,\Phi)} \bigcup_{r \in R} \mathcal{D}(r, \mathbf{i})
\end{array}
$$

where,

$$
\langle {}_{\alpha}[{}_{\gamma}[i,j]_{\delta,\in}[k,l]_{\zeta}]_{\beta}\rangle \triangleq \begin{cases}
\left\{{}_{\alpha}[{}_{\gamma'}[i,j']_{\delta',\in'}[k',l]_{\zeta'}]_{\beta}\right\} & \\
\text{where } & j' = \min(j,l) \\
& k' = \max(k,i) \\
& \gamma' = \gamma \wedge i \neq -\infty \\
& \delta' = \delta \wedge \min(j,l) \neq \infty \wedge (j < l \vee \zeta \wedge \alpha \wedge \beta) \\
& \in' = \in \wedge \max(k,i) \neq -\infty \wedge (k > i \vee \gamma \wedge \beta \wedge \alpha) \\
& \zeta' = \zeta \wedge l \neq \infty \\
\text{when } & i \leq j' \wedge k' \leq l \wedge \\
& [i = j' \rightarrow (\gamma' \wedge \delta')] \wedge [k' = l \rightarrow (\in' \wedge \zeta')] \wedge \\
& [i = l \rightarrow (\alpha \wedge \beta)] \wedge \\
& i \neq \infty \wedge j' \neq -\infty \wedge k' \neq \infty \wedge l \neq -\infty \\
\{\} & \text{otherwise}
\end{cases}
$$

$$
{}_{\alpha_1}[{}_{\gamma_1}[i_1,j_1]_{\delta_1,\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1} \cap {}_{\alpha_2}[{}_{\gamma_2}[i_2,j_2]_{\delta_2,\in_2}[k_2,l_2]_{\zeta_2}]_{\beta_2} \triangleq
$$

$$
\begin{aligned}
&\langle {}_{\alpha_1}[{}_{\gamma}[\max(i_1,i_2),\min(j_1,j_2)]_{\delta,\in}[\max(k_1,k_2),\min(l_1,l_2)]_{\zeta}]_{\beta_1}\rangle \\
&\quad \text{where } \gamma = \begin{cases} \gamma_1 & i_1 > i_2 \\ \gamma_1 \wedge \gamma_2 & i_1 = i_2 \\ \gamma_2 & i_1 < i_2 \end{cases} \\
&\quad \delta = \begin{cases} \delta_1 & j_1 < j_2 \\ \delta_1 \wedge \delta_2 & j_1 = j_2 \\ \delta_2 & j_1 > j_2 \end{cases} \\
&\quad \in = \begin{cases} \in_1 & k_1 > k_2 \\ \in_1 \wedge \in_2 & k_1 = k_2 \\ \in_2 & k_1 < k_2 \end{cases} \\
&\quad \zeta = \begin{cases} \zeta_1 & l_1 < l_2 \\ \zeta_1 \wedge \zeta_2 & l_1 = l_2 \\ \zeta_2 & l_1 > l_2 \end{cases} \\
&\quad \text{when } \alpha_1 = \alpha_2 \wedge \beta_1 = \beta_2 \\
&\{\} \text{ otherwise}
\end{aligned}
$$

$$\neg_{\alpha}[_{\gamma}[i,j]_{\delta},_{\in}[k,l]_{\zeta}]_{\beta} \triangleq \begin{pmatrix} \langle_{\alpha}[_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}},_{\mathrm{T}}[-\infty,k]_{\neg\in}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}},_{\neg\zeta}[l,\infty]_{\mathrm{T}}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{\mathrm{T}}[-\infty,i]_{\neg\gamma},_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{\neg\delta}[j,\infty]_{\mathrm{T}},_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}}]_{\beta}\rangle \cup \\ \langle_{\neg\alpha}[_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}},_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}}]_{\beta}\rangle \cup \\ \langle_{\alpha}[_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}},_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}}]_{\neg\beta}\rangle \cup \\ \langle_{\neg\alpha}[_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}},_{\mathrm{T}}[-\infty,\infty]_{\mathrm{T}}]_{\neg\beta}\rangle \end{pmatrix}$$

$$\textsc{Span}(_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1},_{\alpha_2}[_{\gamma_2}[i_2,j_2]_{\delta_2},_{\in_2}[k_2,l_2]_{\zeta_2}]_{\beta_2}) \triangleq$$

$$\begin{pmatrix} \langle_{\alpha_1}[_{\gamma_1}[i_1,j]_{\delta},_{\in}[k,l_1]_{\zeta_1}]_{\beta_1}\rangle \cup \\ \langle_{\alpha_1}[_{\gamma_1}[i_1,j]_{\delta},_{\in}[k,l_2]_{\zeta_2}]_{\beta_2}\rangle \cup \\ \langle_{\alpha_2}[_{\gamma_2}[i_2,j]_{\delta},_{\in}[k,l_1]_{\zeta_1}]_{\beta_1}\rangle \cup \\ \langle_{\alpha_2}[_{\gamma_2}[i_2,j]_{\delta},_{\in}[k,l_2]_{\zeta_2}]_{\beta_2}\rangle \end{pmatrix}$$

where $j = \min(j_1, j_2)$

$k = \max(k_1, k_2)$

$\delta = [(\delta_1 \wedge j_1 \le j_2) \vee (\delta_2 \wedge j_1 \ge j_2)]$

$\in = [(\in_1 \wedge k_1 \ge k_2) \vee (\in_2 \wedge k_1 \le k_2)]$

$$\mathfrak{D}(=,\mathbf{i}) \triangleq \{\mathbf{i}\}$$

$$\mathfrak{D}(<,_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2,\beta_2\in\{\mathbf{T},\mathbf{F}\}} \langle_{\alpha_2}[_{\neg\beta_1\wedge\neg\alpha_2\wedge\in_1}[k_1,\infty]_{\mathbf{T}},_{\mathbf{T}}[-\infty,\infty]_{\mathbf{T}}]_{\beta_2}\rangle$$

$$\mathfrak{D}(>,_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2,\beta_2\in\{\mathbf{T},\mathbf{F}\}} \langle_{\alpha_2}[_{\mathbf{T}}[-\infty,\infty]_{\mathbf{T}},_{\mathbf{T}}[-\infty,j_1]_{\neg\alpha_1\wedge\neg\beta_2\wedge\delta_1}]_{\beta_2}\rangle$$

$$\mathfrak{D}(\mathsf{m},_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2\in\{\mathbf{T},\mathbf{F}\}} \langle_{\neg\beta_1}[_{\in_1}[k_1,l_1]_{\zeta_1},_{\mathbf{T}}[-\infty,\infty]_{\mathbf{T}}]_{\beta_2}\rangle$$

$$\mathfrak{D}(\mathsf{mi},_{\alpha_1}[_{\gamma_1}[i_1,j_1]_{\delta_1},_{\in_1}[k_1,l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2\in\{\mathbf{T},\mathbf{F}\}} \langle_{\alpha_2}[_{\mathbf{T}}[-\infty,\infty]_{\mathbf{T}},_{\gamma_1}[i_1,j_1]_{\delta_1}]_{\neg\alpha_1}\rangle$$

$$\mathfrak{D}(\mathsf{o}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \alpha_2 \wedge \zeta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \epsilon_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{oi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\alpha_1 \wedge \beta_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{s}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \epsilon_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{si}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\mathbf{T}}[-\infty, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{f}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \rangle$$

$$\mathfrak{D}(\mathsf{fi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, \infty]_{\mathbf{T}}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \rangle$$

$$\mathfrak{D}(\mathsf{d}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \epsilon_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{di}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \overset{\Delta}{=} \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T},\mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

and,

$$\mathcal{I}(\mathbf{i}, r, \mathbf{j}) \overset{\Delta}{=} \bigcup_{\mathbf{i}' \in \mathfrak{D}(r^{-1}, \mathbf{j})} \; \bigcup_{\mathbf{i}'' \in \mathbf{i}' \cap \mathbf{i}} \; \bigcup_{\mathbf{j}' \in \mathfrak{D}(r, \mathbf{i})} \; \bigcup_{\mathbf{j}'' \in \mathbf{j}' \cap \mathbf{j}} \textsc{Span}(\mathbf{i}'', \mathbf{j}'').$$

13. A computer program product embodied in a computer-readable medium for implementing the computation of all occurrences of a compound event from occurrences of primitive events where the compound event is a defined combination of the primitive events, the computer program product comprising:

(a) computer readable program code means for defining primitive event types;

(b) computer readable program code means for defining combinations of the primitive event types as a compound event type;

(c) computer readable program code means for inputting the primitive event occurrences, such occurrences being specified as the set of temporal intervals over which a given primitive event type is true; and

(d) computer readable program code means for computing the compound event occurrences, such occurrences being specified as the set of temporal intervals over which the compound event type is true, wherein the sets of temporal intervals in steps (c) and (d) are specified as smaller sets of spanning intervals, each spanning interval representing a set of intervals.

14. The computer program product according to claim 13, wherein the spanning intervals take the form ${}_{\alpha}[{}_{\gamma}[i, j]_{\delta}, {}_{\in}[k,l]_{\zeta}]_{\beta}$, where $\alpha, \beta, \gamma, \delta, \in,$ and ζ are Boolean values, $i, j, k,$ and l are real numbers, ${}_{\alpha}[{}_{\gamma}[i, j]_{\delta}, {}_{\in}[k,l]_{\zeta}]_{\beta}$ represents the set of all intervals ${}_{\alpha}[p,q]_{\beta}$ where $i \leq_{\gamma} p \leq_{\delta} j$ and $k \leq_{\in} q \leq_{\zeta} l$, ${}_{\alpha}[p,q]_{\beta}$ represents the set of all points r where $p \leq_{\alpha} r \leq_{\beta} q$, and $x \leq_{\theta} y$ means $x \leq y$ when θ is true and $x < y$ when θ is false.

15. The computer program product according to claim 14, wherein the compound event type in (b) is specified as an expression in temporal logic.

16. The computer program product according to claim 15, wherein the temporal logic expressions are constructed using the logical connectives $\forall, \exists, \vee, \wedge_{R}, \Diamond_{R},$ and $\neg$, where R ranges over sets of relations between one-dimensional intervals.

17. The computer program product according to claim 16, wherein the relations are =, <, >, m, mi, o, oi, s, si, f, fi, d, and di.

18. The method according to claim 17, wherein the compound event occurrences are computed using the following set of equations:

$$
\begin{aligned}
\varepsilon(M, p(c_1,\ldots,c_n)) &\triangleq \{\mathbf{i} \mid p(c_1,\ldots,c_n)\,@\,\mathbf{i} \in M\} \\
\varepsilon(M, \Phi \vee \Psi) &\triangleq \varepsilon(M,\Phi) \cup \varepsilon(M,\Psi) \\
\varepsilon(M, \forall x \Phi) &\triangleq \bigcup_{\mathbf{i}_1 \in \varepsilon(M,\Phi[x:=c_1])} \cdots \bigcup_{\mathbf{i}_n \in \varepsilon(M,\Phi[x:=c_n])} \mathbf{i}_1 \cap \cdots \cap \mathbf{i}_n \\
&\quad \text{where } C(M) = \{c_1,\ldots,c_n\} \\
\varepsilon(M, \exists x \Phi) &\triangleq \bigcup_{c \in C(M)} \varepsilon(M, \Phi[x := c]) \\
\varepsilon(M, \neg \Phi) &\triangleq \bigcup_{\mathbf{i}'_1 \in \neg \mathbf{i}_1} \cdots \bigcup_{\mathbf{i}'_n \in \neg \mathbf{i}_n} \mathbf{i}'_1 \cap \cdots \cap \mathbf{i}'_n \\
&\quad \text{where } \varepsilon(M,\Phi) = \{\mathbf{i}_1,\ldots,\mathbf{i}_n\} \\
\varepsilon(M, \Phi \wedge_R \Psi) &\triangleq \bigcup_{\mathbf{i} \in \varepsilon(M,\Phi)} \bigcup_{\mathbf{j} \in \varepsilon(M,\Psi)} \bigcup_{r \in R} \mathcal{I}(\mathbf{i}, r, \mathbf{j}) \\
\varepsilon(M, \Diamond_R \Phi) &\triangleq \bigcup_{\mathbf{i} \in \varepsilon(M,\Phi)} \bigcup_{r \in R} \mathfrak{D}(r, \mathbf{i})
\end{aligned}
$$

where,

$$
\langle {}_{\alpha}[{}_{\gamma}[i,j]_{\delta},{}_{\in}[k,l]_{\zeta}]_{\beta} \rangle \triangleq
\begin{cases}
\left\{ {}_{\alpha}[{}_{\gamma'}[i,j']_{\delta'},{}_{\in'}[k',l]_{\zeta'}]_{\beta} \right\} \\
\text{where } \; j' = \min(j,l) \\
\qquad\quad k' = \max(k,i) \\
\qquad\quad \gamma' = \gamma \wedge i \neq -\infty \\
\qquad\quad \delta' = \delta \wedge \min(j,l) \neq \infty \wedge (j < l \vee \zeta \wedge \alpha \wedge \beta) \\
\qquad\quad \in' = \in \wedge \max(k,i) \neq -\infty \wedge (k > i \vee \gamma \wedge \beta \wedge \alpha) \\
\qquad\quad \zeta' = \zeta \wedge l \neq \infty \\
\text{when } \; i \leq j' \wedge k' \leq l \wedge \\
\qquad\quad [i = j' \rightarrow (\gamma' \wedge \delta')] \wedge [k' = l \rightarrow (\in' \wedge \zeta')] \wedge \\
\qquad\quad [i = l \rightarrow (\alpha \wedge \beta)] \wedge \\
\qquad\quad i \neq \infty \wedge j' \neq -\infty \wedge k' \neq \infty \wedge l \neq -\infty \\
\{\} \qquad \text{otherwise}
\end{cases}
$$

$$
{}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \cap {}_{\alpha_2}[{}_{\gamma_2}[i_2, j_2]_{\delta_2}, {}_{\in_2}[k_2, l_2]_{\zeta_2}]_{\beta_2} \triangleq
$$

$$
\langle {}_{\alpha_1}[{}_{\gamma}[\max(i_1, i_2), \min(j_1, j_2)]_{\delta}, {}_{\in}[\max(k_1, k_2), \min(l_1, l_2)]_{\zeta}]_{\beta_1} \rangle
$$

$$
\text{where } \gamma = \begin{cases} \gamma_1 & i_1 > i_2 \\ \gamma_1 \wedge \gamma_2 & i_1 = i_2 \\ \gamma_2 & i_1 < i_2 \end{cases}
$$

$$
\delta = \begin{cases} \delta_1 & j_1 < j_2 \\ \delta_1 \wedge \delta_2 & j_1 = j_2 \\ \delta_2 & j_1 > j_2 \end{cases}
$$

$$
\in = \begin{cases} \in_1 & k_1 > k_2 \\ \in_1 \wedge \in_2 & k_1 = k_2 \\ \in_2 & k_1 < k_2 \end{cases}
$$

$$
\zeta = \begin{cases} \zeta_1 & l_1 < l_2 \\ \zeta_1 \wedge \zeta_2 & l_1 = l_2 \\ \zeta_2 & l_1 > l_2 \end{cases}
$$

when $\alpha_1 = \alpha_2 \wedge \beta_1 = \beta_2$

{} otherwise

$$
\neg_{\alpha}[{}_{\gamma}[i, j]_{\delta}, {}_{\in}[k, l]_{\zeta}]_{\beta} \triangleq \begin{pmatrix} \langle {}_{\alpha}[{}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, k]_{\neg\in}]_{\beta} \rangle \cup \\ \langle {}_{\alpha}[{}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}, {}_{\neg\zeta}[l, \infty]_{\mathrm{T}}]_{\beta} \rangle \cup \\ \langle {}_{\alpha}[{}_{\mathrm{T}}[-\infty, i]_{\neg\gamma}, {}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}]_{\beta} \rangle \cup \\ \langle {}_{\alpha}[{}_{\neg\delta}[j, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}]_{\beta} \rangle \cup \\ \langle {}_{\neg\alpha}[{}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}]_{\beta} \rangle \cup \\ \langle {}_{\alpha}[{}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}]_{\neg\beta} \rangle \cup \\ \langle {}_{\neg\alpha}[{}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}, {}_{\mathrm{T}}[-\infty, \infty]_{\mathrm{T}}]_{\neg\beta} \rangle \end{pmatrix}
$$

$$\textsf{SPAN}\left({}_{\alpha_1}\left[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}\right]_{\beta_1}, {}_{\alpha_2}\left[{}_{\gamma_2}[i_2, j_2]_{\delta_2}, {}_{\in_2}[k_2, l_2]_{\zeta_2}\right]_{\beta_2}\right) \triangleq$$

$$\begin{pmatrix} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j]_{\delta}, {}_{\in}[k, l_1]_{\zeta_1}]_{\beta_1} \rangle \cup \\ \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j]_{\delta}, {}_{\in}[k, l_2]_{\zeta_2}]_{\beta_2} \rangle \cup \\ \langle {}_{\alpha_2}[{}_{\gamma_2}[i_2, j]_{\delta}, {}_{\in}[k, l_1]_{\zeta_1}]_{\beta_1} \rangle \cup \\ \langle {}_{\alpha_2}[{}_{\gamma_2}[i_2, j]_{\delta}, {}_{\in}[k, l_2]_{\zeta_2}]_{\beta_2} \rangle \end{pmatrix}$$

where $j = \min(j_1, j_2)$

$k = \max(k_1, k_2)$

$\delta = [(\delta_1 \wedge j_1 \leq j_2) \vee (\delta_2 \wedge j_1 \geq j_2)]$

$\in = [(\in_1 \wedge k_1 \geq k_2) \vee (\in_2 \wedge k_1 \leq k_2)]$

$$\mathfrak{D}(=, \mathbf{i}) \triangleq \{\mathbf{i}\}$$

$$\mathfrak{D}(<, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\neg\beta_1 \wedge \neg\alpha_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}, {}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(>, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}, {}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \neg\beta_2 \wedge \delta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{m}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\neg\beta_1}[{}_{\in_1}[k_1, l_1]_{\zeta_1}, {}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{mi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, \infty]_{\mathbf{T}}, {}_{\gamma_1}[i_1, j_1]_{\delta_1}]_{\neg\alpha_1} \rangle$$

$$\mathfrak{D}(\mathsf{o}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \alpha_2 \wedge \zeta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{oi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\alpha_1 \wedge \beta_2 \wedge \gamma_1}[i_1, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{s}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \in_1}[k_1, \infty]_{\mathbf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{si}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\beta_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\mathbf{T}}[-\infty, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{f}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2 \in \{\mathbf{T}, \mathbf{F}\}} \langle {}_{\alpha_2}[{}_{\mathbf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\in_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \rangle$$

$$\mathfrak{D}(\mathsf{fi}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2 \in \{\mathsf{T},\mathsf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, \infty]_{\mathsf{T}}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1} \rangle$$

$$\mathfrak{D}(\mathsf{d}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathsf{T},\mathsf{F}\}} \langle {}_{\alpha_2}[{}_{\mathsf{T}}[-\infty, j_1]_{\neg\alpha_1 \wedge \alpha_2 \wedge \delta_1}, {}_{\neg\beta_1 \wedge \beta_2 \wedge \epsilon_1}[k_1, \infty]_{\mathsf{T}}]_{\beta_2} \rangle$$

$$\mathfrak{D}(\mathsf{di}, {}_{\alpha_1}[{}_{\gamma_1}[i_1, j_1]_{\delta_1}, {}_{\epsilon_1}[k_1, l_1]_{\zeta_1}]_{\beta_1}) \triangleq \bigcup_{\alpha_2, \beta_2 \in \{\mathsf{T},\mathsf{F}\}} \langle {}_{\alpha_2}[{}_{\alpha_1 \wedge \neg\alpha_2 \wedge \gamma_1}[i_1, \infty]_{\mathsf{T}}, {}_{\mathsf{T}}[-\infty, l_1]_{\beta_1 \wedge \neg\beta_2 \wedge \zeta_1}]_{\beta_2} \rangle$$

and,

$$\mathcal{I}(\mathbf{i}, r, \mathbf{j}) \triangleq \bigcup_{\mathbf{i}' \in \mathfrak{D}(r^{-1}, \mathbf{j})} \bigcup_{\mathbf{i}'' \in \mathbf{i}' \cap \mathbf{i}} \bigcup_{\mathbf{j}' \in \mathfrak{D}(r, \mathbf{i})} \bigcup_{\mathbf{j}'' \in \mathbf{j}' \cap \mathbf{j}} \textsc{Span}(\mathbf{i}'', \mathbf{j}'').$$